Representational Structure of Numbers in Children : Analyses of Strategies in Solving both Addition and Subtraction Problems[1]

Kazuhiro KURIYAMA and Hajime YOSHIDA[2]

幼児の数表象の構造

── 加減算問題におけるストラテジー分析 ──

栗 山 和 広 ・ 吉 田 甫

Abstract

Representation of preschool children's number concepts was investigated in the present study. Children were given both addition and subtraction problems. Four counting strategies were found for all the addition and subtraction problems. The strategies were overt counting (O-type), covert counting (C-type), direct counting (D-type), and internal counting (I-type). The frequency ratio of direct counting type for addition and subtraction problems increased significantly for problems with numbers below 5 more than problems with numbers over 5. Children used more advanced strategies in solving problems with numbers below 5. These results suggested that preschool children represent numbers to 5 as a firm structure or privileged anchor for numbers.

Young children have been said to lack understanding of conservation, seriation, class inclusion, and many other concepts. Recently, however, a number of researches have noted the limitations of this approach and suggested that it is more revealing to focus on the capabilities that preschoolers do have. One area in which this suggestion has lead to extensive research is a representation of the structure of number concepts in children (Ginsburg, 1977 ; Greeno, Riley, & Gelman, 1984 ; Riley, Greeno, & Heller, 1983).

It is clear that the adult structure of the number concepts is clearly composed of the decimal number system. Attempts have just begun to elaborate the development of structures of number concepts (Resnick, 1983). Resnick divides the development of the understanding of number representation into three broad periods. In this period children construct a representation of numbers that can be appropriately characterized as a mental number line. The second is found in the early primary period. In this period children can interpret numbers in terms of part-whole relationships and understand numbers as compositions of other numbers. The third appears in the later primary period. This period is further divided into three substages during which children's representation of number concepts is modified to reflect knowledge of the decimal structure of the counting and notational systems.

This paper is interested in considering whether or not there is some structure of number concepts in younger children. Siegler and Robinson (1982) found some evidence that pre-school children have a representation of the decade structure. They asked children to produce a word number sequence and examined the place in the sequence at which word production stopped. They found that the three most common stopping points were 29, 39, and 49. A similar point of view has been reported by Fuson, Richards, and Briars (1982). By examining the children's number words, Fuson et al. found that children jumped entire decades, for example, from 17, 18, 19 to 30 or 40. These results were regarded as a reflection of the children's konwledge of the decade structure.

These results do not necessarily guarantee that children have a representation of the decade structure for every number. Siegler and Robinson (1982) and Fuson et al. (1982) found evidence for such representation of numbers over 20. They did not, however, refer to numbers below 20 in their reports. Therefore, we have no idea whether or not there is some structure to the numbers below 20. Ginsburg (1977) has argued that the first 12 numbers are completely arbitrary in younger children. Resnick's theory on the numbers below 10 is similar to this one. She assumes that a representation of numbers in preschoolers can be characterized as a mental number line in which numbers are linked to each other by the next relationships. Many theories, therefore, commonly assume that younger children have no such representation as the decade structure to the numbers below 10, in particular.

Previous studies, however, do not necessarily confirm this assumption. Gelman and Gallistel (1978) asked children to count the objects on the cards. However, over half of the three- and four-year old children accurately judged the number of objects within five. If the children's representation of number concepts is like the one depicted by Resnick (1983), we would have predicted no difference in accuracy among the numbers of the objects. Because Gelman and Gallistels' results do not confirm such a prediction, it may be possible that the children's structure of number concepts may not merely be a simple mental number line as Resnick (1983) assumed.

Yoshida and Kuriyama (1986) were examined the possibility of whether or not there is some structure for numbers below 20. Several experiments confirmed the hypothesis that pre-school children may represent numbers to 5 as a privileged anchor for the number below 10. For example, children were asked to resolve numbers into 5's and x's or to find supplements to 10. It was significantly easier and faster for children to resolve numbers than to find supplements. These results supported the presence of the representation system of the numbers to 5 as a firm structure or privileged anchor.

The purpose of the present study was to test whether or not preschool children may represent numbers to 5 as a firm structure or privileged anchor, by examining strategies used in both addition and subtraction tasks. If children have such a representation of numbers concepts, they should have different strategies for solving problems with numbers less than 5 compared to those with numbers greater than 5.

Method

Subjects. The subjects were 22 children (14 boys and 8 girls) attending a private kindergarten in Miyazaki City. The mean age of the subjects was 5 ; 9. They came primary from middle class homes.

Materials. Both addition and subtraction problems were used in this experiment. In the addition problems both the augend and addend were single digits. The sums of the addition problems were 10 or less. Problems involving carrying were excluded. Twenty-four addition problems were selected. All met the criteria, including the one problem in the practice phase. These problems were presented in such a way that whenever number was larger it was always the augend. Both the minuend and subtrahend of subtraction problems were 9 or less. Problems whose answers 0 (e. g., 4-4 or 7-7) were excluded. Thirty-five subtraction problems were used. All met the criteria, including the one problem in the practice phase. This experiment used video cameras and video tape recorders to record the activities of the children in solving the addition and subtraction problems.

Procedure. Each child was run individually. All children received practice and test phase for the addition problems followed by practice and test phases for the subtraction problems. The experimenter told them that today's game was playing with numbers. She told the children to use their fingers in responding to the questions. The children were also asked to place their hands on the table.

The practice addition problems were presented in the following manner. There are two brothers, Ken and Sho. Their mother gave one chocolate to Ken and one to Sho. How many chocolates did Ken and Sho have all together ? Some feedback was given to a child by the experimenter using her fingers. In the test phase the child was given 24 addition problems. Subsequent to the test phase of the addition problems, the practice subtraction problem (2-1) was presented to the child. In the subtraction test phase the child was given 35 problems.

The time required to finish the experiment ranged from 45 to 90 minutes for each child. The experiment was divided into several sessions depending upon the physical and emotional states of the child. Each session was run for about 30 minutes per day. The video camera was set to the left front of the child and recorded all the activities of the child during the experiment. No feedback was given at any time except for assurances that the child was doing well.

Results and Discussion.

Performance on both addition and subtraction problems. As the number of the problems was different for addition and subtraction, the scores for accuracy were determined by using the ratio of the number of correct answers to the total number of addition and subtraction problems. The mean ratios of correct responses for the addition and subtraction problems were. 84 and. 83, respectively. A t-test showed that there were no significant performance differences between addition and subtraction problems. However, previous studies (Fujinaga et al., 1963; Starkey & Gelman, 1982) have reported that subtraction was more difficult than addition for younger children. Of course, as the intention of these studies was not to compare performance on addition with that on subtraction, they used a small number of problems. We think, therefore, that our results are more reliable than these previous studies.

Analyses of strategies. We examined the children's strategies in solving both addition and subtraction problems by analyzing the video tapes recorded in the experiment. The counting strategies of the children could be divided into one of the four types. The first was overt counting in which children counted numbers by opening their fingers one by one overtly (O-type). The second type was covert counting. In this type children counted by opening their fingers covertly, by following their fingers with their eyes, or by moving their heads slightly (C-type). The third type was a direct representation of numbers. Children represented numbers on their fingers directly without counting their fingers one by one or covertly (D-type). The last was a type in which we were not able to observe any visible strategy. When questioned, children who had used this type responded that they had done the problems in their heads. Therefore, this type was called internal representation (I-type).

The strategies for all the addition and subtraction problems were analyzed as combinations of these four counting types. In the addition problems, we found twelve strategies. Ten of them are shown in Table 1. The remaining two strategies were excluded from subsequent analysis because their frequency was very low (the frequency of both O-C and I-D was one each). The first letter of each strategy indicates the strategy which was used in processing the augend and the second letter the strategy which was used for the addend. A strategy of D-D in the problem 6+2, for example, means that the children first opened their six fingers directly without overt or covert counting, and then added two fingers directly to the six fingers, before responding 8. The other strategies can be explained similarly.

The first column on the left of Table 1 indicates the ratios of each strategy to the total number of problems, because children used one or more kinds of strategies in the addition problems. As can be seen in Table 1, the most frequent strategy was D-D. Additional

Table 1 Ratios of Frequency and Proportion Correct for Ten Strategies and Ratios of Frequency in Sub-problems with Numbers Below and Over 5 in Addition

Strategy	All problem		Sub-problems	
	Ratios	Proportion correct	Below 5	Over 5
D-D	.420(33.08)	.730(37.26)	.548(41.33)	.328(33.52)
I-I	.225(32.93)	.472(47.76)	.230(32.93)	.201(33.52)
D-I	.135(24.47)	.458(47.51)	.103(22.55)	.072(12.85)
C-C	.076(17.33)	.153(32.94)	.071(17.48)	.096(21.90)
O-D	.030(7.30)	.090(28.74)	.0 (0)	.100(20.44)
D-O	.028(6.49)	.101(26.40)	.013(4.70)	.057(11.74)
O-O	.026(8.50)	.186(35.96)	.0 (0)	.045(11.17)
C-I	.024(8.82)	.086(27.35)	.019(7.51)	.027(5.37)
D-C	.016(3.40)	.136(34.31)	.010(3.35)	.027(5.37)
O-C	.012(3.65)	.090(28.74)	.003(1.47)	.036(11.88)

Note. Below 5 means problems with numbers below 5. Over 5 means problems with numbers over 5. Standard deviations are in parentheses.

strategies with 5% or greater frequency were I-I, D-I, and C-C. In other words, children used direct internal strategies more often than overt ones in solving addition problems. The proportions correct for these strategies were about 50 % as seen in the second column of Table 1. When children relied on covert counting for the addition problems, they solved the problems poorly.

A somewhat different tendency was observed for the strategies used in solving the subtraction problems. We found a total of fourteen strategies. Because the frequency of four of them was 2 or less, these strategies were excluded from subsequent analysis. Table 2 shows the ratio of each strategy to the total number of problems and the proportions correct for subtraction problems. There were seven kinds of strategies with 5 % or greater frequency. This means that the children adopted a greater variety of strategies for subtraction than for addition. Such differences were clarified by conducting a t-test between the addition and subtraction problems. There was a significant decrease in the frequency ratio of I-I from addition to subtraction, $t(21)=2,19$, $p<.05$. On the other hand, there were significant increases in the frequency ratios of D-O, D-C, and O-D strategies, $t(21)=3.52$, $p<.01$; $t(21)=3.02$, $p<.01$; $t(21)=2.94$, $p<05$, respectively.

Furthermore, children may have adopted different strategies for solving problems with numbers less than 5 compared with those numbers greater than 5. Therefore, we again analyzed the data of Table 1 and 2. All the addition and subtraction problems were divided into two categories. The first contained problems which had numbers of 5 or less than 5 in either or both terms. The ratios of each strategy to the total number of problems in each

Table 2 Ratios of Frequency and Proportion Correct for Ten Strategies and Ratios of Frequency in Sub-problems with Numbers Below and Over 5 in Subtraction

	All problems		Sub-problems	
Strategy	Ratios	Proportion correct	Below 5	Over 5
D-D	.312(28.34)	.648(41.52)	.496(32.57)	.284(29.01)
D-O	.136(23.42)	.369(44.41)	.140(26.00)	.152(22.14)
I-I	.127(26.23)	.260(39.28)	.149(28.50)	.105(25.23)
D-C	.090(12.75)	.415(43.63)	.090(15.92)	.086(13.96)
D-I	.083(12.75)	.480(48.33)	.105(17.08)	.083(12.61)
O-D	.081(14.82)	.252(41.46)	.0 (0)	.092(19.19)
O-O	.074(15.48)	.369(45.49)	.016(5.59)	.099(18.25)
C-C	.046(12.56)	.130(33.01)	.020(6.38)	.054(15.52)
C-I	.038(10.32)	.151(33.26)	.020(9.24)	.038(10.30)
O-C	.013(2.51)	.272(44.53)	.0 (0)	.012(2.91)

Note. Below 5 means problems with numbers below 5. Over 5 means problems with numbers over 5. Standard deviations are in parentheses.

category are also shown in Table 1 for addition problems and in Table 2 for subtraction problems. The frequency ratio of D-D for addition problems increased significantly for problems with numbers below 5 compared to those with numbers over 5, $t\,(21)=3.49$, $p<.01$. There were, on the other hand, significant decreases in the frequency ratios of O-D and D-O for addition problems with numbers below 5 compared to those with numbers over 5, $t\,(21)=2.24$, $p<.05$; $t\,(21)=2.11$, $p<.05$, respectively. For subtraction problems, the ratio of D-D increased significantly for problems with numbers below 5 compared to those with numbers over 5, $t\,(21)=3.05$, $p<.01$. However, there were significant decreases in the ratios of O-D and O-O for problems with numbers below 5 compared to those with numbers over 5, $t\,(21)=2.21$, $p<.05$; $t\,(21)=2.52$, $p<.01$, respectively.

These results indicate that children relied less on direct counting strategies for problems with numbers over 5 compared to problems with numbers below 5. Instead, they relied on more primitive strategies in solving problems with numbers over 5. In general, the counting strategies would develop from the overt to the internal through direct or covert strategies. Therefore, we could say children are able to manage the numbers 1 to 5 very easily. From these results of counting strategies it was suggested that preschool children represent as a firm structure or priviliged anchor for the numbers below 10.

A problem concerns the nature of what is represented as the priviliged anchor in the number concept of children. So far in this research, we have referred to the priviliged anchor as the numbers 1 to 5. However, we may be able to refer not only to the numbers 1 to 5 but also just to the number 5. Can we distinguish the former from the latter ? The latter involves no other numbers. For example, if the priviliged anchor contains all the numbers to 5, does the number 4 have a function similar to the findings obtained in the present experiments? Siegler and Robinson (1982) and Miller and Gelman (1983) found three clusters among the numbers below 10. The first was the numbers 1 to 3. The second was the numbers 4 and 5. These are only data which we know concering this problem. Therefore, further research may focus attention on this problem.

Resnick (1983) assumed that a mental number line existed in which numbers below 10 are linked by next relations to each other. However, our present research suggests that there is some structure for the numbers below 10. Such a conclusion is worth further study. For example, do elementaly school children who are instructed in the decade structure of numbers also have the representational system of the numbers to 5 as the priviliged anchor? Further research should also center on this question.

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